Licenciatura Management

Operational Research Chapter 1

2017-2018

ISEG 100 1911-2011 ANOS

100 ANOS A PENSAR NO FUTURO

Teacher

Name: Leonor Santiago Pinto

e-mail: lspinto@iseg.utl.pt

Office: 506 Quelhas Building

Bibliography

- F.S. Hillier, G.J. Lieberman, *Introduction to Operations Research*, 9th edition, McGraw-Hill, International Edition, New York, 2010;
- M.C. Mourão, L. Santiago Pinto, O. Simões, J. Valente, M.V. Pato, *Investigação Operacional: Exercícios e Aplicações*, 1ª edição, Verlag Dashöfer, Lisboa, 2011 (in portuguese)

Assessment Process

- During the break week there will be a midterm exam that covers the first half of the program. In the "época Normal" (EN) there will be a second midterm that covers the second half of the program. In this day students who wish may choose to repeat the evaluation of the first half of the program. Each half of the program has a 50% worth of the final grade .
- Students who do not get a grade higher than or equal to 9.5 in the average of the two mini-tests or examination of EN, or a minimum of (8 in 20, in each mid term) will be submitted to the "época de Recurso" .
- Students who obtain 17.5 or more and would like to have a final grade greater than 17 may be called to an oral exam.
- It is allowed to consult 1 A4 sheet in the midterms and 2 A4 sheets in the other exams .
- Calculating machines are not allowed, neither in the exams nor in the midterms.
- All that is not specified above follows the "Regime Geral de Avaliação de Conhecimentos".

Course contents:

1. Linear Programming
2. The Simplex Method
3. Duality and Sensitivity Analysis
4. The Transportation and the Assignment Problems
5. Network Optimization
6. Integer Linear Programming

Objectives of the course:

The objective of this course is to introduce the students to the wide field of applications for (integer) linear programming and network models, as well as provide basic knowledge of the respective mathematical models.

Students will be required to apply very simple algorithms and dominate the resolution of (integer) linear programming problems with the Solver/Excel software. Special emphasis will be given to the economic interpretation of results.

Detailed program:

1. Linear Programming (LP)

1.1 Introduction

1.2 Formulation and Graphical Solution

1.3 Definitions and Properties

1.4 Solving Problems by Solver/Excel

2. Simplex Method

2.1 Introduction

2.2 Augmented Form and Basic Feasible Solutions

2.3 Simplex Algorithm

3. Duality and Sensitivity Analysis

3.1 Introduction

3.2 Duality

3.3 Economic Interpretation of Duality. Shadow Prices. Primal-Dual Relations

3.4 Sensitivity Analysis

- Changes in the Right-Hand Sides of the Constraints
- Changes in the Coefficients of the Objective Function

4. Transportation and Assignment Problems

4.1 Introduction

4.2 Transportation Problem

4.3 Assignment Problem

5. Network Optimization

5.1 Introduction

5.2 Minimum Cost Flow Problem

5.3 Shortest-Path Problem

5.4 Minimum Spanning Tree Problem

- Prim Algorithm

6. Integer Linear Programming (ILP)

6.1 Introduction

6.2 Integer Linear Programming Problems

6.3 Graphical and Solver/Excel Solution

6.4 Formulations with Binary Variables

1. Linear Programming (LP)

1.1 Introduction

1.2 Formulation and Graphical Solution

1.3 Definitions and Properties

1.4 Solving Problems by Solver/Excel

Prototype Example 1

x_1 – no. batches of P1 produced per week (P1=8-foot glass door with aluminum framing)

x_2 – no. batches of P2 produced per week (P2=4×6 foot double-hung wood framed window)

Z – total profit per week (in thousands of dollars) from producing these two products

Linear Programming (LP) Model:

$$Max\ Z = 3x_1 + 5x_2$$

$$s.t.\begin{cases} x_1 \leq 4 \\ 2x_2 \leq 12 \\ 3x_1 + 2x_2 \leq 18 \\ x_1, x_2 \geq 0 \end{cases}$$

LP model – standard form

$$Z^* = Max\ Z = \sum_{j=1}^{n} c_j x_j$$ Objective Function (OF)

$$\text{s.t.} \begin{cases} \sum_{j=1}^{n} a_{ij} x_j \le b_i & i = 1, 2, \dots, m \\ x_j \ge 0 & j = 1, 2, \dots, n \end{cases}$$

Functional Constraints
Sign Constraints

Decision variable: $x_j \ (j = 1, \dots, n)$ represents level of activity *j*

Data:

c_j coefficient on the objective function of the decision variable *j*;

b_i righ-hand-side (RHS) of the functional constraint *i*;

a_{ij} technical coefficient of the decision variable *j* on the functional constraint *i.*

c_j,b_i and a_{ij} are called the parameters of the LP model

Linear Programming

Assumptions of Linear Programming

Proportionality: The contribution of each activity (j) to the value of the objective function and to the left-hand-side of the constraints is proportional to the level of the activity (x_j).

Additivity: The value of the objective function and the value of the left-hand-side of the constraints are the sum of the individual contributions of the various activities.

Divisibility: The variables assume real values ($x_j \in R$).

Certainty: Every coefficient (also called parameter) is assumed to be a known constant.

Linear Programming

Definitions I

Solution of an LP – a vector of R^n which components are the values of the variables;

Feasible Solution (**FS**) – a solution that satisfies all the constraints (functional and sign);

Non Feasible Solution (**NFS**) – a solution that does not satisfy at least one of the constraints;

Feasible Region (**FR**) – the set of all feasible solutions;

Optimal Solution (**OS**) – a feasible solution that gives the best value to the objective function (OF) (the best value=maximum or minimum);

Optimal value – the value of the objective function at an optimal solution;

Binding constraint in a solution – a constraint that hold with equality at that solution;

To solve an LP is to determine the optimal solution (or solutions) and the optimal value or to conclude that an optimal solution does not exist and why.

Linear Programming

Prototype 2

$x_1 -$ no. units of TV advertisement (unit=standart block of advertisement P&G)

$x_2 -$ no. units of advertisement in print media

$$Min\ z = x_1 + 2x_2$$

$$\text{s.t.} \begin{cases} x_2 \geq 3 & \text{(R1)} \\ 3x_1 + 2x_2 \geq 18 & \text{(R2)} \\ -x_1 + 4x_2 \geq 4 & \text{(R3)} \\ x_1, x_2 \geq 0 \end{cases}$$

a) $Max\ z = x_1 + 2x_2$

$$\text{s. t.}\begin{cases} x_1 - 2x_2 \leq 3 \\ x_1 + x_2 \leq 3 \\ x_1, x_2 \geq 0 \end{cases}$$

b) $Max\ z = 3x_1 + 4x_2$

$$\text{s. t.}\begin{cases} x_1 - 2x_2 \geq 4 \\ x_1 + x_2 \leq 3 \\ x_1, x_2 \geq 0 \end{cases}$$

c) $Max\ z = x_1 + x_2$

$$\text{s. t.}\begin{cases} x_1 - x_2 \leq 2 \\ x_1 - x_2 \geq 0 \\ x_1, x_2 \geq 0 \end{cases}$$

d) $Max\ z = x_1 - x_2$

$$\text{s. t.}\begin{cases} x_1 - x_2 \leq 2 \\ x_1 - x_2 \geq 0 \\ x_1, x_2 \geq 0 \end{cases}$$

e) $Max\ z = -10x_1 - 5x_2$

$$\text{s. t.}\begin{cases} x_1 - x_2 \leq 5 \\ x_1 + \frac{8}{5}x_2 \geq -3 \\ x_1 \text{free} \\ x_2 \leq 0 \end{cases}$$

h) $min\ z = x_1 + x_2$

$$\text{s. t.}\begin{cases} x_1 - x_2 \leq 2 \\ x_1 - x_2 \geq -2 \\ x_1, x_2 \geq 0 \end{cases}$$

j) $Max\ z = 3x_1 + 6x_2$

$$\text{s. t.}\begin{cases} x_1 + 2x_2 \leq 4 \\ x_1 - x_2 \geq 0 \\ x_1, x_2 \geq 0 \end{cases}$$

LP – solving by solver of excel – prototype example 1

data

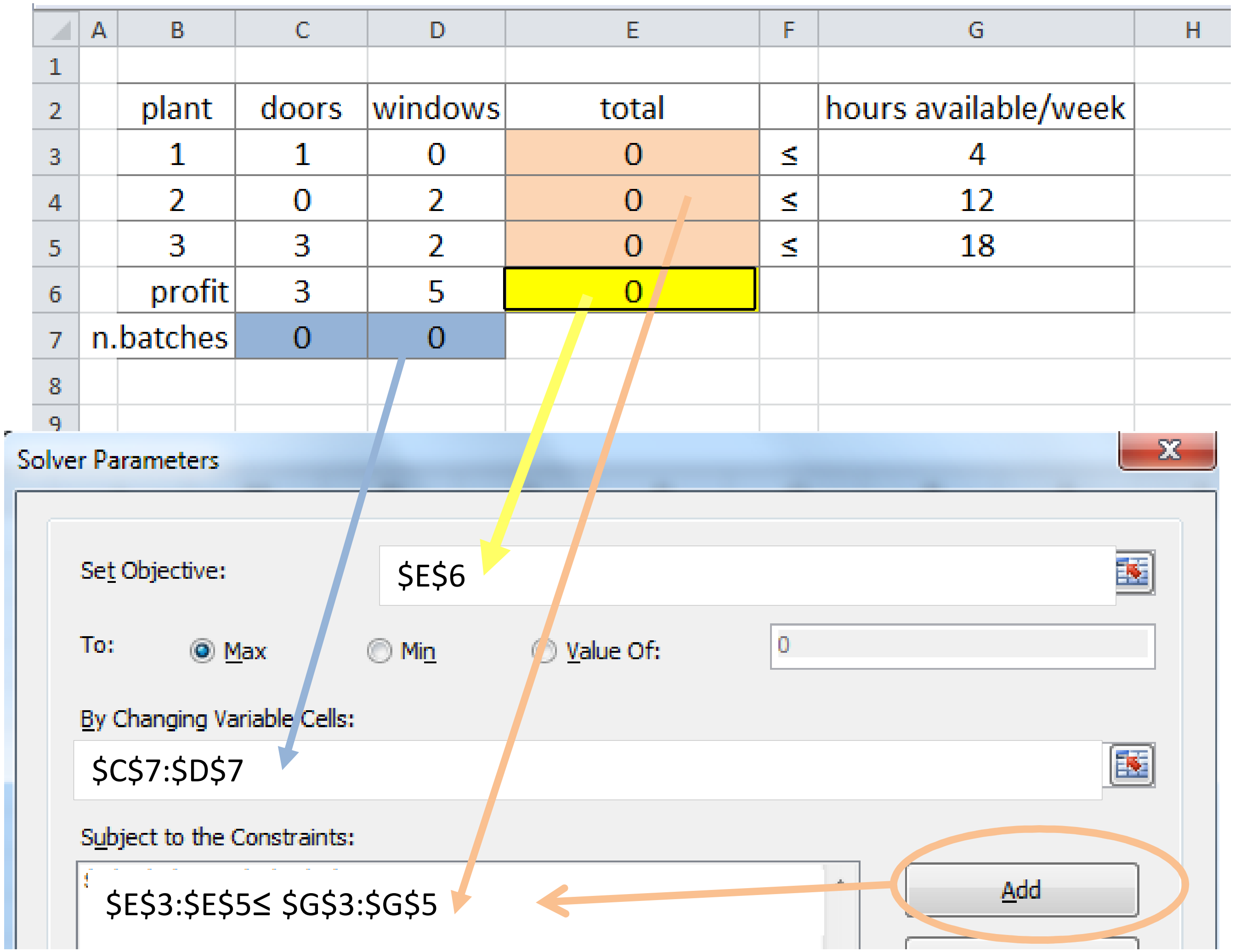
data

	A	B	C	D	E	F	G	H
1								
2		plant	doors	windows	total		hours available/week	
3		1	1	0	0	≤	4	
4		2	0	2	0	≤	12	
5		3	3	2	0	≤	18	
6		profit	3	5	0			
7	n.batches		0	0				
8								
9								

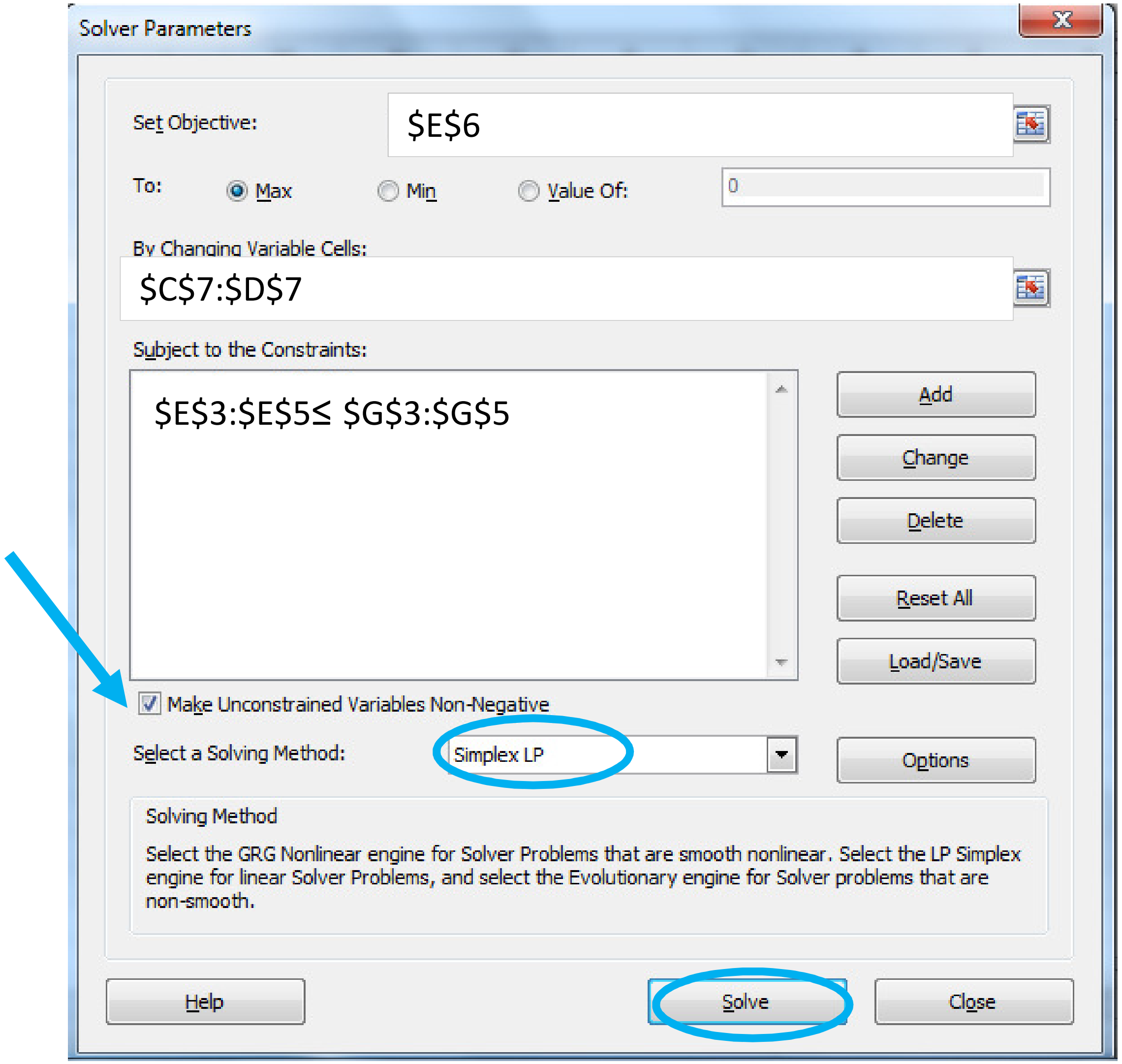
initial values

	E
1	
2	total
3	=SUMPRODUCT(C3:D3;$C$7:$D$7)
4	=SUMPRODUCT(C4:D4;$C$7:$D$7)
5	=SUMPRODUCT(C5:D5;$C$7:$D$7)
6	=SUMPRODUCT(C6:D6;$C$7:$D$7)
7	

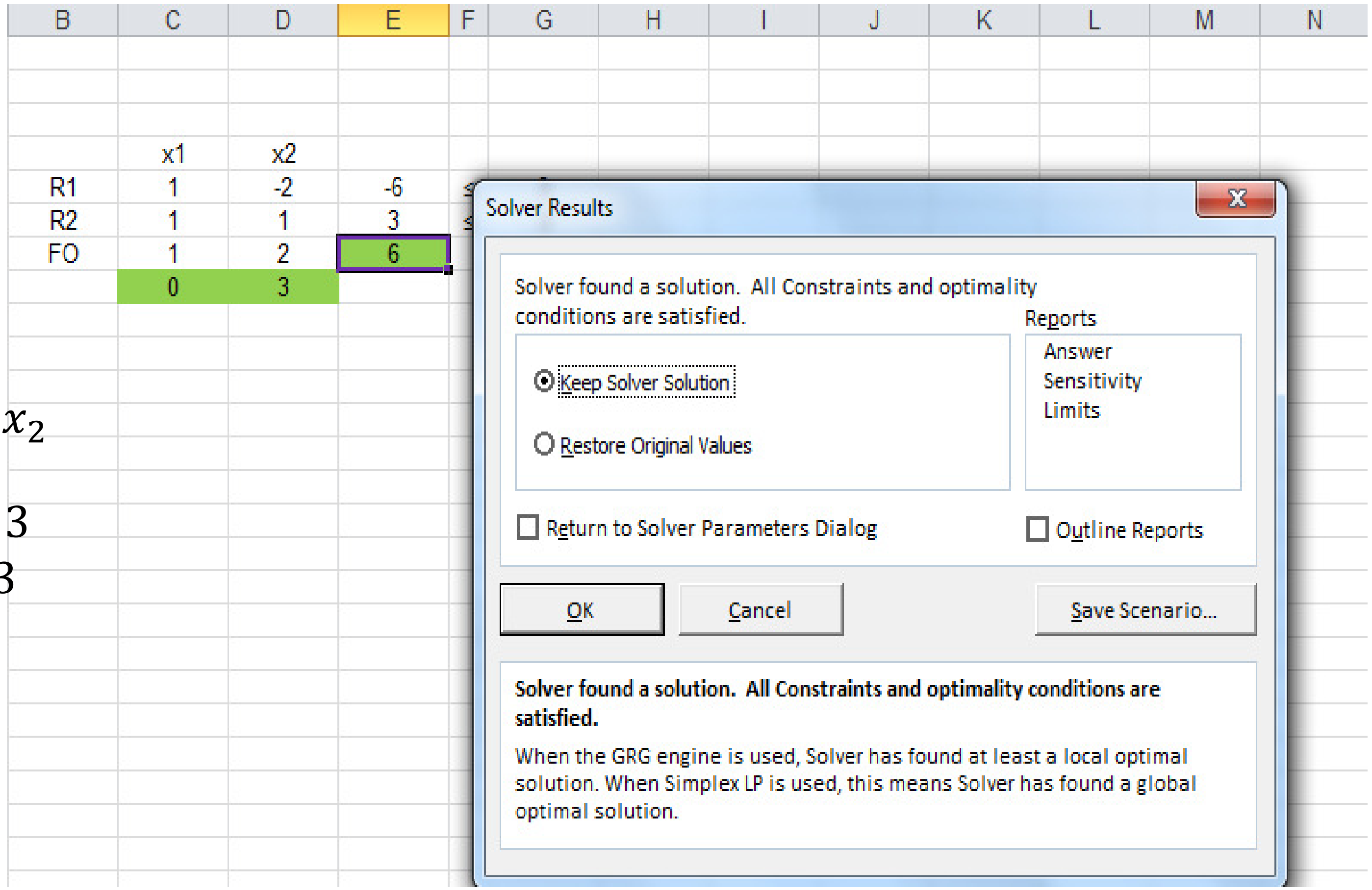
LP – solving by solver of excel – prototype example 1



LP – solving by solver of excel – prototype example 1



1 a)

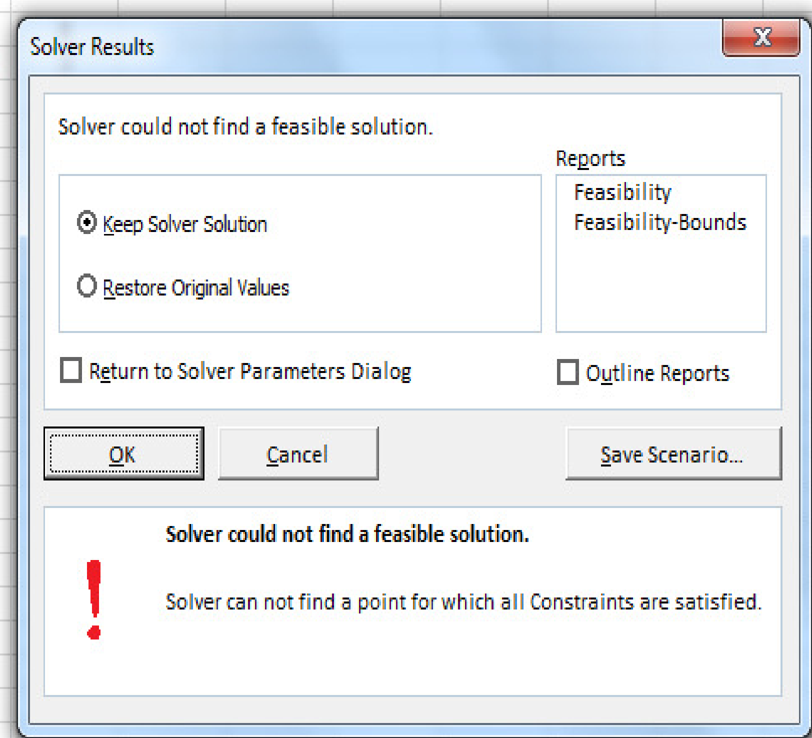


$$Max\ z = x_1 + 2x_2$$

$$\text{s. t.}\begin{cases} x_1 - 2x_2 \leq 3 \\ x_1 + x_2 \leq 3 \\ x_1, x_2 \geq 0 \end{cases}$$

1 b) infeasible

	x1	x2		
R1	1	-2	3	≥
R2	1	1	3	≤
FO	3	4	9	
	3	0		



$$Max\ z = 3x_1 + 4x_2$$

$$\text{s. t.} \begin{cases} x_1 - 2x_2 \geq 4 \\ x_1 + x_2 \leq 3 \\ x_1, x_2 \geq 0 \end{cases}$$

1 c) unbounded

	x1	x2			
R1	1	-1	2	≤	2
R2	1	-1	2	≥	0
FO	1	1	2		
	2	0			

$$Max\ z = x_1 + x_2$$

$$\text{s. t.} \begin{cases} x_1 - x_2 \leq 2 \\ x_1 - x_2 \geq 0 \\ x_1, x_2 \geq 0 \end{cases}$$